

ELECTROSTATES

Properties of charges -

- Basic property of matter
- charge without mass can not exist whereas mass without charge can exist.
- * - Quantization of charge -
charge on a body can only exist in the form of 'e'

$$Q = ne \quad (n = \text{integer})$$

- * - charge is additive in nature

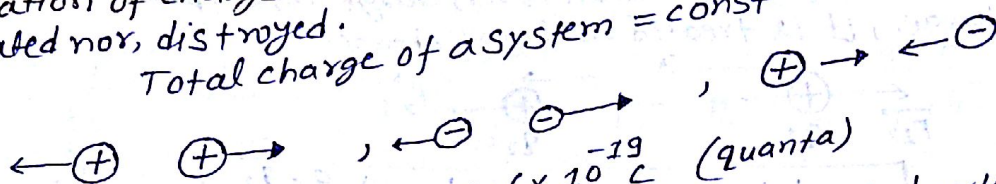
$$Q_{\text{net}} = q_1 - q_2 + q_3 - q_4$$

$$q_1, -q_2$$

$$q_3, -q_4$$

- * - conservation of charge - charge on an isolated system can neither be created nor, destroyed.
Total charge of a system = const.

NOTE →



- Minimum possible charge $e = 1.6 \times 10^{-19} \text{ C}$ (quanta)
- Exception of quantisation -

quark
→ up $(+\frac{2e}{3})$
→ down $(-\frac{e}{3})$

$$\begin{cases} 2u + 1d = 1 \text{ particle} \\ 2(+\frac{2e}{3}) + 1(-\frac{e}{3}) = +e \\ 1u + 2d = 1 \text{ neutron} \\ (\frac{2e}{3}) + 2(-\frac{e}{3}) = \text{zero} \end{cases}$$

- quark particle don't exist independently, so quantisation is still correct.
- * - If quark particle would exist even then quantisation would be valid quanta will be $(e/3)$.
- In a conductor charge is distributed at outer surface only while in non-conductor charge is distributed inside the surface.

METHOD OF CHARGING -

i) → Friction -

⊕ve ⇒ glass rod, dry hair, cat skin, wool.
⊖ve ⇒ silk, comb, Ebonite, plastic/Amber.

Ex → cloud charging, charging of oil drop in miliken oil drop experiment.

iii) → conduction -

* For ⊕ve charge will move [High → Low]
* For ⊖ve charge will move [Low → High]

* NOTE ⇒ In conduction total charge of system is re-distributed in the ratio of radius for making potential same.
* After conduction potential become same while charges will differ.

iiii) → Induction - takes place in facing layer only.

$$Q_{\text{Induced}} = Q_{\text{Inducing}} \left(1 - \frac{1}{\epsilon_r}\right) \quad (\epsilon_r \rightarrow \text{dielectric const. of body})$$

* For metal ($\epsilon_r = \infty$) → $Q_{\text{Induced}} = Q_{\text{Inducing}}$

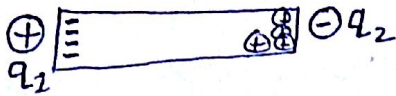
* For Non-metal ($\epsilon_r \neq \infty$) → $Q_{\text{Induced}} < Q_{\text{Inducing}}$.

⇒ * Best method of charging.

- NOTE** →
- * Induction affect the distribution of charge not the magnitude of charge.
 - * There will be attraction b/w neutral charge body.
 - * There will be attraction b/w body having charge of same nature provided that magnitude of charges will be different.
 - * Sure test of charging is repulsion not attraction.

EX → How will the force ~~on~~ on q_1 will change if an insulated rod is kept b/w them as shown?

Ans → Force will ↑



COLUMB'S LAW -

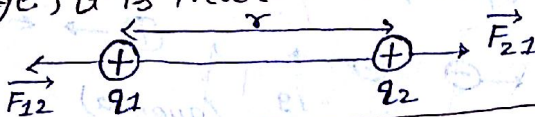
q_1 ← r → q_2

$F \propto \frac{1}{r^2}$, $F \propto \frac{q_1 q_2}{r^2}$

$$F = \frac{k q_1 q_2}{r^2}$$

$k = 9 \times 10^9 \text{ (MKS)}$
 $k = 1 \text{ (cgs)}$

* If the distance in discussion is very large as compared with the dimension of charge, it is treated as point charge.



$$|\vec{F}_{21}| = |\vec{F}_{12}| = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{q_1 q_2}{r^2}$$

ϵ_0 → permittivity of vacuum or, free space = $8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

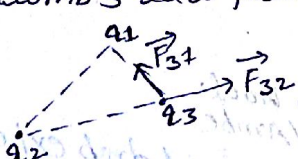
$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

III NOTE

* If the charges are kept in some other medium, permittivity = $\epsilon_0 \epsilon_r$
 ϵ_r = Relative permittivity or, dielectric const. of medium will ↓

$$|\vec{F}_{net}| = \left(\frac{1}{4\pi\epsilon_0 \epsilon_r} \right) \frac{q_1 q_2}{r^2}$$

* coulomb's law follow principle of superposition.



$$\vec{F}_{3net} = \vec{F}_{31} + \vec{F}_{32}$$

AJMS

* PERMITTIVITY [E]

- Permittivity of vacuum (ϵ_0) ⇒ $\epsilon_0 = 8.85 \times 10^{-12} C^2 / N \cdot m^2$
- Permittivity of medium
- Absolute permittivity of medium (E)
unit → $C^2 / N \cdot m^2$
- $\epsilon_r = \frac{E}{\epsilon_0}$

$$1 \leq \epsilon_r < \infty$$

$(\epsilon_r)_{air} = 1$
 $(\epsilon_r)_{metal} = \infty$

$$F_{vacume} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$$

$$F_{medium} = \frac{1}{4\pi\epsilon_0 \epsilon_r} \times \frac{q_1 q_2}{r^2}$$

$$\therefore F_{medium} = \frac{F_{vacume}}{\epsilon_r}$$

$\because \epsilon_r > 1$
 $\Rightarrow F_{net} < F_{vacume}$

11) → case

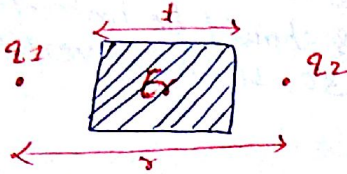
If Fair = Fmedium

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{air}^2} = \frac{1}{4\pi\epsilon_0 \epsilon_r} \cdot \frac{q_1 q_2}{r_{med}^2}$$

$$r_{air}^2 = \epsilon_r r_{med}^2$$

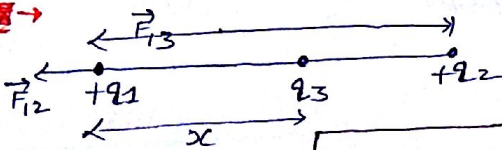
$$r_{air} = \sqrt{\epsilon_r \cdot r_{med}}$$

12) → case



$$F_{\text{partial medium}} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{\left\{ (r-x) + t\sqrt{\epsilon_r} \right\}^2}$$

Imp concept
IIT



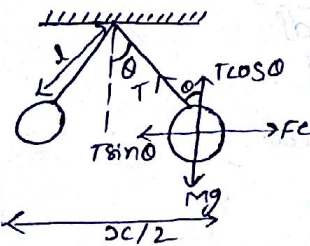
There should be third charge q_3 placed in b/w, so that net force on all the charges become zero, Also find value of q_3 .

$$x = \frac{d\sqrt{q_1}}{\sqrt{q_1+q_2}} = \frac{d}{\left(1 + \sqrt{\frac{q_2}{q_1}}\right)}$$

$$q_3 = \frac{-q_1 q_2}{(\sqrt{q_1} + \sqrt{q_2})^2}$$

concept
#

Two identical charge simple pendulum are in equilibrium as shown in fig. (Given θ = small)



iii) → calculate Repulsion b/w ball

$$\tan\theta = \frac{kq^2}{x^2 mg}$$

$$x = \left\{ \frac{2kq^2 l}{mg} \right\}^{1/3}$$

2016 AIPMT
AIEEE 2012
iii) → If charge of ball start to leave at const. rate & ball are moving towards each other with velocity (v) then Relation b/w x & v ?

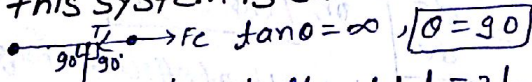
$$q^2 \propto x^3 \quad \left| \quad \frac{dq}{dt} \propto \frac{3}{2} x^{1/2} \frac{dx}{dt} \right. \quad \text{AIPMT 2016}$$

$$v \propto x^{-1/2}$$

iiii) → If $q_1 < q_2 \Rightarrow \theta_1 = \theta_2$ (due to Action Reaction pair).

liv) → If $M_1 > M_2 \Rightarrow \theta_1 < \theta_2$ (M_1 is heavy so it will displace less so angle less)

vi) → If this system is carried in space or, Artificial satellite. [$g=0$]



- * Separation b/w ball = $l + l = 2l$
- * Angle b/w string = $90^\circ + 90^\circ = 180^\circ$
- * Electric force b/w both = $F_e = \frac{kq^2}{(2l)^2} = \frac{kq^2}{4l^2}$
- * Tension in each string = $F_e = \frac{kq^2}{4l^2}$

JIPMER 2016

If we want to give charge on ball in due to repulsion ~~not~~ string become horizontal or, 180° then the value of q = ?

$$T = \frac{kq_1 q_2}{(2l)^2}, \quad q = \sqrt{\frac{4l^2 mg}{k}}$$

$$q = \sqrt{\frac{4l^2 T}{k}} \quad [T=mg]$$

2012 AIEEE

vi) → If this system is dipped into the liq & angle of string with verticle remain unchanged find dielectric cost. of liq. (density = 1.6 g/cm^3 , density liq = 0.8 g/cm^3)

$$\epsilon_r = \frac{1}{1 - \frac{d}{db}} = \frac{1}{1 - \frac{0.8}{1.6}} = \frac{1}{\frac{1.6 - 0.8}{1.6}} = \frac{1}{\frac{0.8}{1.6}} = \frac{1}{\frac{1}{2}} = 2$$

$$= 2$$

concept

Two identical balls each with density ' ρ ' are suspended with a common point by two insulating strings of equal length. Both the balls have equal mass & charge. In equilibrium the string makes an angle ' θ ' with vertical. Now the whole system is emerged in liquid with density ' σ '. If angle ' θ ' does not change, what is dielectric const of liquid.

$$\epsilon_r = \frac{\rho}{\rho - \sigma}$$

concept

Three identical small balls each with mass ' m ' are suspended at one point by threads of length ' l '. What charges should be imparted to the ball for each thread to form an angle 30° with the vertical.

$$q = l \sqrt{\pi \epsilon_0 m g}$$

concept

A small charge ' $+q$ ' is distributed uniformly on an insulating ring of Radius ' R '. If an additional charge ' $+Q$ ' is kept at centre, find increment in tension of in Ring.

$$T = \frac{k Q q}{2 \pi R^2}$$

continuous charge distribution: $\rightarrow$

|| $\rightarrow$ Linear charge density (λ) $\rightarrow$ charge per unit length.

* If distribution is uniform $\rightarrow \lambda = \frac{q}{l}$

* If distribution is non uniform $\Rightarrow \lambda = \frac{dq}{dx}$ or, $\frac{dq}{dl}$

$$dq = \lambda dx$$

||| $\rightarrow$ Surface charge density (σ) $\rightarrow$ charge per unit area.

* If distribution is uniform $\rightarrow \sigma = \frac{q}{A}$

* If the distribution is non-uniform $\rightarrow \sigma = \frac{dq}{dA}$

$$dq = \sigma dA$$

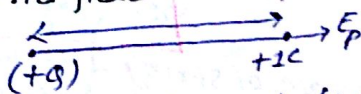
||| $\rightarrow$ Volume charge density (ρ) $\rightarrow$ charge per unit volume.

* If ' ρ ' is uniform $\rightarrow \rho = \frac{q}{V}$

* If ' ρ ' is non-uniform $\rightarrow \rho = \frac{dq}{dv}$ or, $dq = \rho dv$

ELECTRIC FIELD INTENSITY (E) $\Rightarrow$ It represents strength of effect of charge at given point.

* Electric field due to point charge: -



$$|\vec{F}_p| = \left(\frac{1}{4 \pi \epsilon_0} \right) \frac{(+Q)(1C)}{r^2} = |\vec{E}|$$

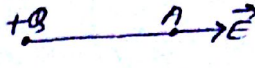
$$|\vec{E}| = \left(\frac{1}{4 \pi \epsilon_0} \right) \frac{Q}{r^2} \frac{N}{C} \text{ or } \frac{V}{m}$$

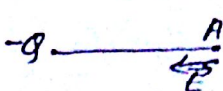
$$E = \frac{kQ}{r^2}$$

NOTE → * When electric field is measured due to a point charge (Q), the test charge taken very small & another definition can be given as

$$|\vec{E}| = \lim_{q_0 \rightarrow 0} \frac{|\vec{F}|}{q_0}$$

* Direction of Electric field: -

⊕ve charge = Away from the charge ($E = \frac{kQ}{r^2}$) 

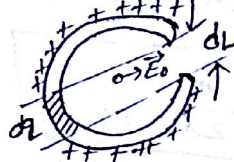
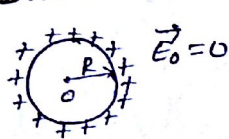
⊖ve charge = Towards the charge ($E = \frac{kQ}{r^2}$) 

* If there are several no of charges in the system then, net Electric field is vector sum of all Electric field due to charges. (principle of superposition)

* If a charge ' q ' is kept in a external Electric field $\vec{E}$, then net force acting on q is $\vec{F} = q\vec{E}$
 [Along $\vec{E}$, If $q \rightarrow \oplus$ ve]
 [Opp $\vec{E}$, If $q \rightarrow \ominus$ ve]

ELECTRIC FIELD DUE TO CONTINUOUS CHARGE DISTRIBUTION: →

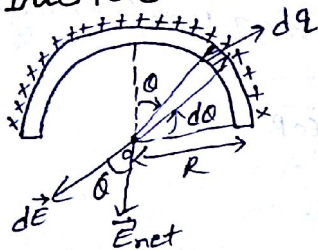
CASE I → Due to uniformly charge Ring of Radius ' R ' at the centre: -



$$\lambda = \frac{Q}{2\pi R}, \quad dq = dL = \frac{Q dL}{2\pi R}$$

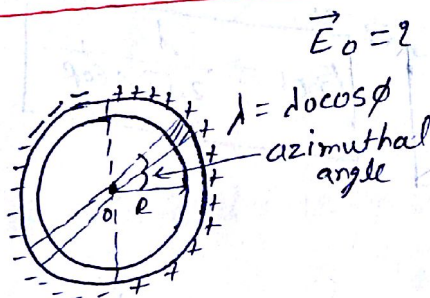
$$|\vec{E}_0| = \frac{1}{4\pi\epsilon_0 R} \frac{dQ}{R^2}$$

CASE II → Due to semi-circular Ring at the centre: -



$$\vec{E}_{net} = \frac{Q}{2\pi^2\epsilon_0 R^2} = \frac{\lambda}{2\pi\epsilon_0 R}$$

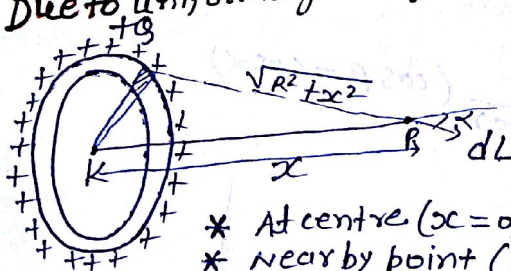
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$$E = \frac{\lambda_0}{2\epsilon_0 R}$$

$$E_{net} = 2E = \frac{\lambda_0}{4\epsilon_0 R}$$

CASE III → Due to uniformly charged circular Ring along its axis: -



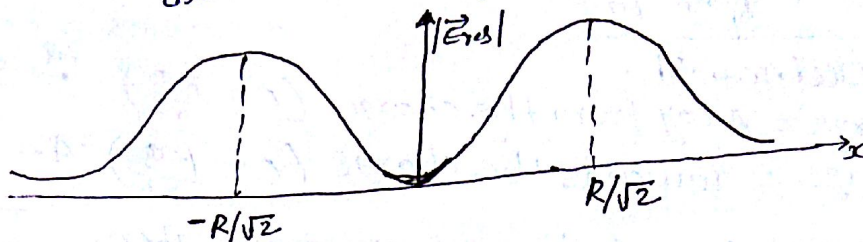
$$E_{res} = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(R^2 + x^2)^{3/2}}$$

- * At centre ($x=0$) $E=0$
- * Near by point ($x \ll R$) $E = \frac{kQx}{(R^2)^{3/2}} = \frac{kQx}{R^3} \propto x$
- * Far away point ($x \gg R$) $E = kQ/x^2 \propto 1/x^2$
- * For $E_{max} = \frac{dE}{dx} = 0 \Rightarrow x = \pm R/\sqrt{2}$

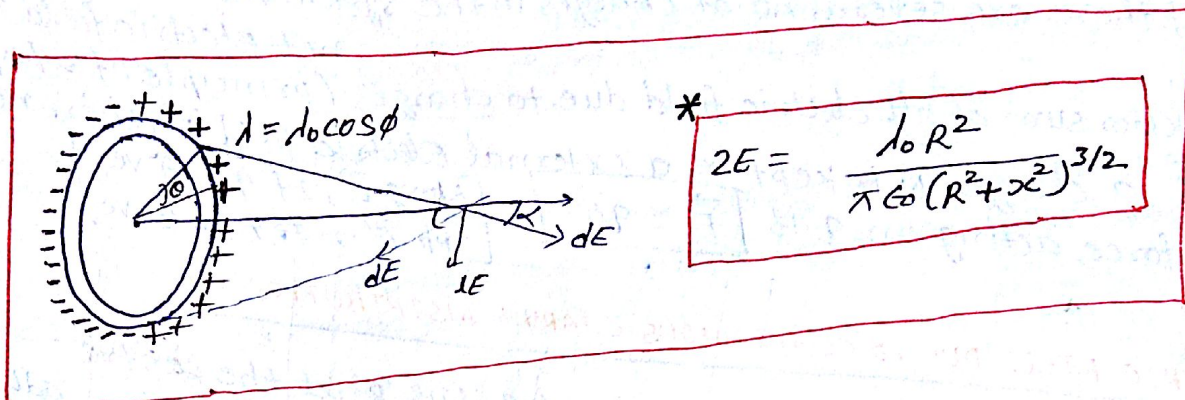
NOTE →

* Max^m electric field along axis

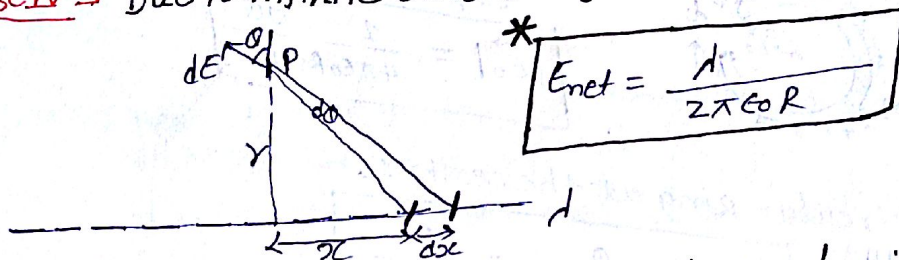
$$\frac{dE}{dx} = 0 \Rightarrow x = \frac{R}{\sqrt{2}}$$



*



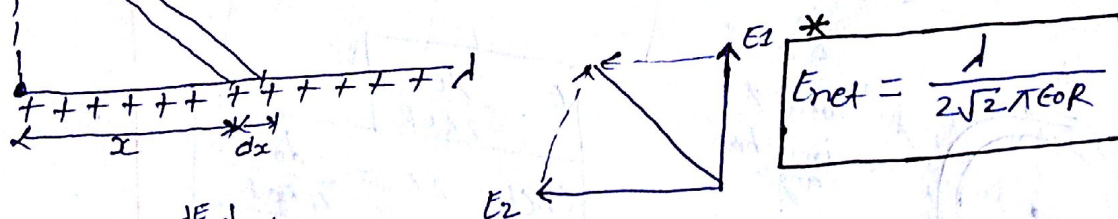
case IV → Due to infinite line charge having uniform charge density λ :-



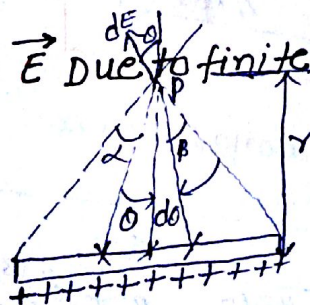
case V → Due to semi-infinite line charge having uniform density λ :-

$$E_1 = \int_0^\infty dE \cos\theta = \frac{\lambda}{4\pi\epsilon_0 R}$$

$$E_2 = \int_0^\infty dE \sin\theta = \frac{\lambda}{4\pi\epsilon_0 R}$$



case VI → $\vec{E}$ Due to finite line charge :-
 $E_p = ?$

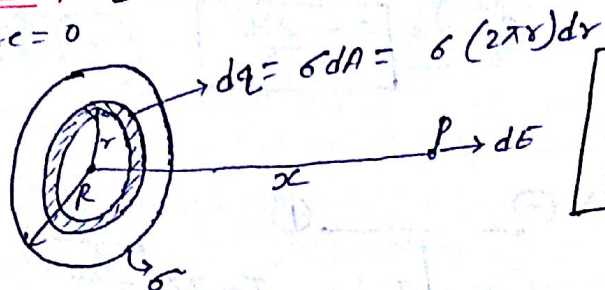


$$E_y = \frac{\lambda}{4\pi\epsilon_0 x} (\sin\beta + \sin\alpha)$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0 y} (\cos\beta - \cos\alpha)$$

Case VII $\Rightarrow \vec{E}$ due to circular DISC uniformly charge With (σ) :-

$\vec{E}_{\text{centre}} = 0$



$$\vec{E} = \frac{\sigma x}{2\epsilon_0} \left[\frac{1}{x} - \frac{1}{\sqrt{R^2 + x^2}} \right]$$

NOTE $\rightarrow$

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \right]$$

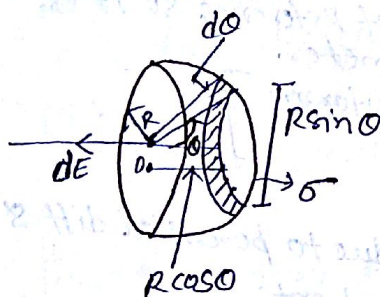
If $R \rightarrow \infty$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}$$

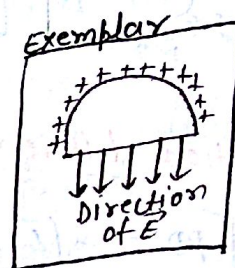
For infinitely distributed plane sheet.

* After folding the Rings, net electric field will $\uparrow$.

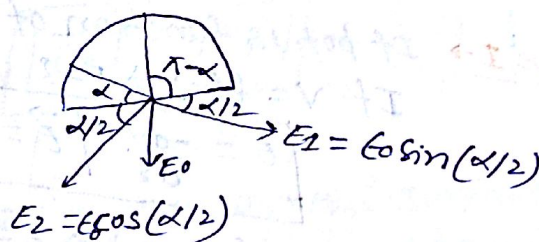
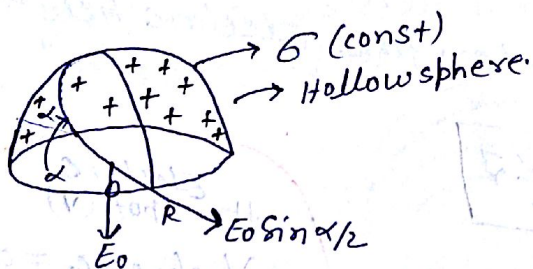
Case VIII $\Rightarrow \vec{E}$ at centre due to uniformly charged hemi-spherical shell :-



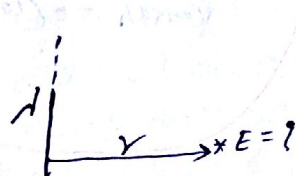
$$E = \frac{\sigma}{4\epsilon_0}$$



EX $\rightarrow$ Find $\vec{E}$ due to part which is cut at an angle α .



* Electric field due to uniformly charge wire -



$$E = \frac{2K\lambda}{r}$$

* $\vec{E}$ centre for uniformly charged arc :-

III $\rightarrow$



$$E = \frac{2K\lambda}{r} \sin\left(\frac{\alpha}{2}\right)$$

iii) $\alpha = 90^\circ$ $E = \frac{\sqrt{2} K \lambda}{r}$ $\alpha = 180^\circ$ $E = \frac{2 K \lambda}{r}$ $\alpha = 360^\circ$ $E = 0$

* Pendulum concept :-

$T = 2\pi \sqrt{\frac{l}{g_{\text{eff}}}}$
 $(g_{\text{eff}} = \text{accln for net force except tension})$

$T = 2\pi \sqrt{\frac{l}{g_{\text{eff}}}}$
 $T = 2\pi \sqrt{\frac{l}{g + \frac{qE}{m}}}$

$F_{\text{net}} = \sqrt{(mg)^2 + (qE)^2}$
 $g_{\text{eff}} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$
 $T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$

Potential difference (PD) :- It is independent of reference so it is absolute parameter.
 $\left[\begin{aligned} \text{PD} &= -\int \vec{E} \cdot d\vec{r} \text{ (If } E \text{ is uniform / non-uniform)} \\ \text{PD} &= -\vec{E} \cdot d\vec{r} \text{ (If } E \text{ is uniform)} \end{aligned} \right]$

Relation b/w E & V :- Electric field is due to potential diff & it is equal to $\ominus$ ve of potential gradient.

Type I $\rightarrow$ If pot. is function of ' r ' then find Electric field :-
 If $V = f(r) \Rightarrow E = ?$

$E = -\frac{dV}{dr} \quad \vec{E} = -\frac{dV}{dr} \hat{r}$

Type II $\rightarrow$ If $V = f(x, y, z) \Rightarrow E = ?$

$\vec{E} = -\nabla V$
 $\nabla \rightarrow$ Del operator (gradient)
 $\nabla = \frac{i\partial}{\partial x} + \frac{j\partial}{\partial y} + \frac{k\partial}{\partial z}$

$\vec{E} = -\left[i \frac{\partial V}{\partial x} + j \frac{\partial V}{\partial y} + k \frac{\partial V}{\partial z} \right]$

Type III $\rightarrow$ If V - r graph is given at $E = ?$

$E = -\frac{dV}{dr} \quad E = -\text{slope}$

Electric pot (V)

$V_{\text{reference}} = 0$
 $V_{\text{location}} = 0$
 $V_{\text{earth}} = -0.12 \text{ V}$
 $V_{\text{earth}} = 0 \text{ (consider zero)}$

$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$
 $E = -\text{slope}$

Type IV → If $E = f(r) = PD = ?$

$$PD = -\int \vec{E} \cdot d\vec{r}$$

Type V → If $E = f(x, y, z) \Rightarrow PD = ?$

$$PD = \int \vec{E} \cdot d\vec{r}$$

$$= - \int (E_x i + E_y j + E_z k) \cdot (dx i + dy j + dz k)$$

$$= - \int E_x dx - \int E_y dy - \int E_z dz$$

Type VI → If $E \cdot r$ graph is given

$$PD = \left| - \int \vec{E} \cdot d\vec{r} \right|$$

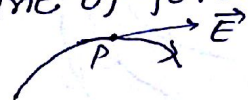
= Area under the curve.

Electric line of forces or, Electric field lines:-

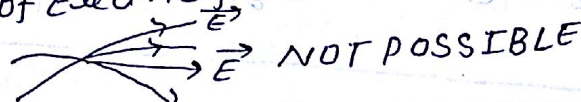
We know that electric field is invisible to generate the picture of Electric field, we draw Electric line of forces.

properties of ELF →

iii → Tangent drawn at a point on the line of forces gives the direction of field at that point.

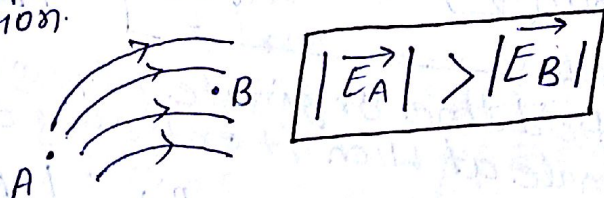


iii → Two electric field lines can never intersect as there will be two direction of Electric field at that point.



iii → conservative Electric field lines can never form close loop.

iv → The density of electric field lines in a certain region gives us a qualitative idea about the strength of field in that region.

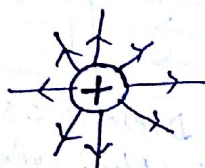


v → They are always perpendicular to equipotential surface.

vi → If the positive charge at rest is free to move then it may or, may not follow the line of forces.

vii → Electric lines are differentiable at all point they can't have sharp turnings.

viii → Electric field lines can be discontinuous.

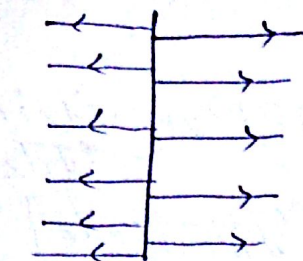


Isolated $\oplus$ ve charge originates at charge & end at infinity.

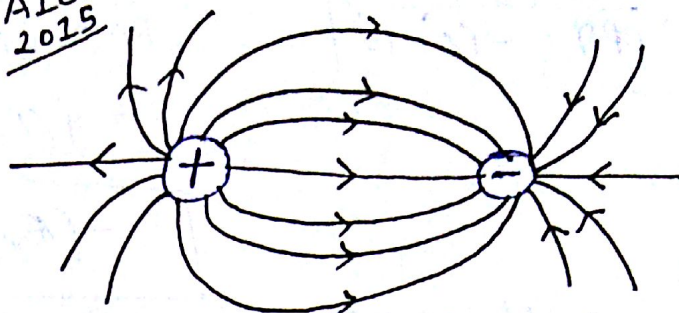


Isolated $\ominus$ ve charge originates at ∞ & end at $\ominus$ ve charge.

AIEEE
2015

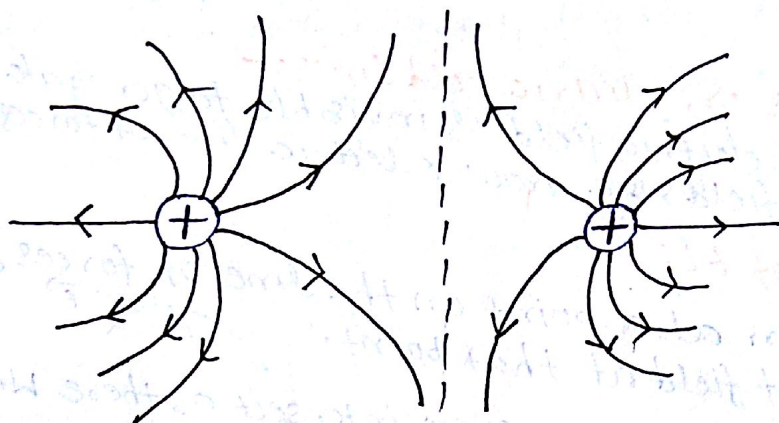


Infinite plane
Sheet of charge



Equal $\oplus$ ve & $\ominus$ ve charge.

*



Equal $\oplus$ ve charges

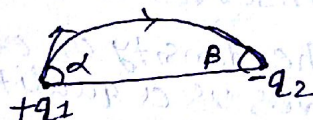
* Equidistance || line
represent uniform
Electric field.

$$E \propto \frac{1}{\text{separation b/w lines}}$$

III

Imp property :-

$$\sqrt{\frac{q_1}{q_2}} \sin\left(\frac{\alpha}{2}\right) = \sin\left(\frac{\beta}{2}\right)$$

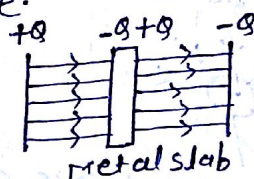


EX $\rightarrow$ If a field line originate at $+2q$ at angle 30° . Find the angle at which it enters $-q$ charge.

$$\sqrt{\frac{2q}{q}} \sin 30^\circ = \sin\left(\frac{\beta}{2}\right) \quad \left| \quad \frac{\beta}{2} = 45^\circ \right.$$

$$\sqrt{2} \times \frac{1}{2} = \sin\left(\frac{\beta}{2}\right) \quad \left| \quad \boxed{\beta = 90^\circ} \right.$$

||X| $\rightarrow$ In a conductor there is no line of force while in non-conductor few line are there.

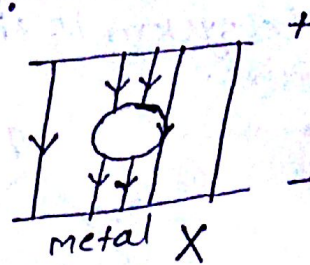
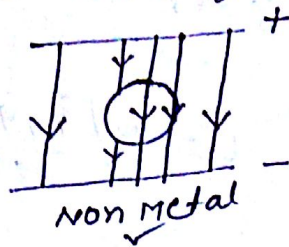
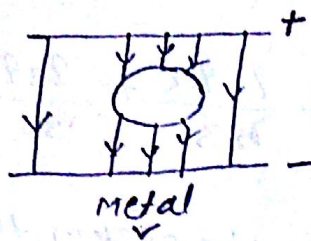


metal slab
Applied $\rightarrow$ Right
Induced $\rightarrow$ Left
 $E_{ind} = E_{app}$
 $E_{net} = 0$

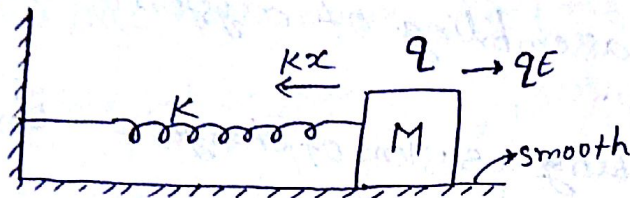


Dielectric slab
Applied $\rightarrow$ Right
Induced $\rightarrow$ Left
 $E_{ind} < E_{app}$
 $E_{net} \neq 0 (< E_{app})$

(X) → Electric at ELF are always to conducting surface this can make any angle with non-conducting surface.



concept
EX →



An Electric field is switched on at $t=0$ as shown.

1a) → amplitude of oscillation *

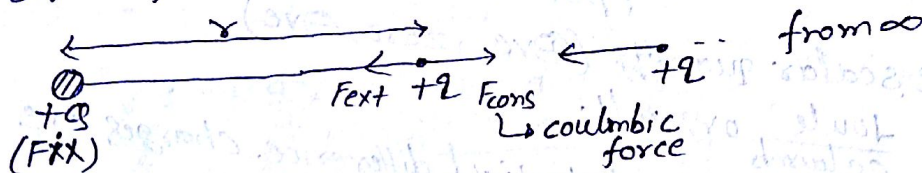
$$Kx = qE \quad \boxed{x = \frac{qE}{K}}$$

1b) → Time period of oscillation.

$$\boxed{T = 2\pi \sqrt{\frac{m}{K}}}$$

ELECTRIC POTENTIAL ENERGY (U): -

Defined for conservative field only.



* point charge → PE = zero.

* If we consider displacement $\vec{dr}$ at position vector $\vec{r}$

$$dW_{\text{cons}} = \vec{F} \cdot \vec{dr}$$

$$\boxed{\vec{F} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Qq}{r^2} \cdot \vec{r}}$$

$$\boxed{U = \frac{Qq}{4\pi\epsilon_0 r}}$$

NOTE

iii) → Q, q are kept along with sign as If Q, q both are of same sign

$W_{\text{ext}} = \oplus \text{ve}$, $U = \oplus \text{ve}$ (unstable)

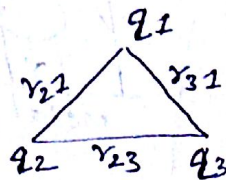
* AS If Q, q are of opp. sign

$W_{\text{ext}} = \ominus \text{ve}$

$U = \ominus$ (stable)

iii) → If there are 'n' no. of point charges then the total energy of the system is the sum of potential energies due to all these pairs.

Ex →



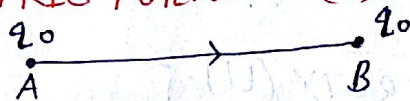
$$U = k \left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}} \right]$$

iiii) → If there are 'n' no. of charges (point) in the system then no. of pair = $\boxed{\frac{n(n-1)}{2}}$

iv) → External Work done in assembling of a system of charges = U_{system} .

Work done in disassembling a system of charges = $-U_{\text{system}}$ = Binding Energy of system.

***// # ELECTRIC POTENTIAL (V) :-** Work done on an unit $\oplus$ ve charge in displacing it from A to B.



$$V_{AB} = V_B - V_A = \frac{W_{\text{ext}}}{q_0} = \frac{V_B - V_A}{q_0} = (\text{pot of B} - \text{pot. of A})$$

NOTE → * It is scalar quantity ($\oplus$ ve, zero, $\ominus$ ve)

* unit $\frac{\text{Joule}}{\text{Coulomb}}$ or, volt.

* While calculating potential difference charges are used along with sign.

Absolute potential at a point :- If we bring charge q_0 from infinity to a point P. then work done per unit charge become potential at P.



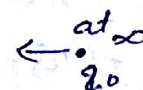
$$V_P = V_P - V_{\infty} = \frac{U_P - U_{\infty}}{q_0} = \frac{U_P}{q_0}$$

Electric potential due to a system of charges —

iii) → Due to point charge (Q) —

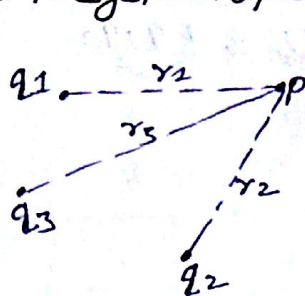


$$V_A = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$



(Q with sign)

iii) → Due to system of several point charges -



$$V_p = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \right]$$

$q_1, q_2, q_3 \Rightarrow$ With sign

Due to continuous charge distribution -

ii) → Due to circular Ring (+Q) →

|a| → At centre

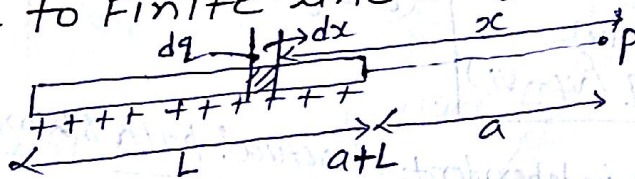
$$V = \frac{kQ}{R}$$

* (Whether this dist. is uniform or, non-uniform)

|b| → At a point on axis

$$V = \frac{kQ}{\sqrt{R^2 + x^2}}$$

*** iii) → Due to Finite line charge -



$$V = k \frac{dq}{x} = \int_0^{a+L} k \lambda \frac{dx}{x}$$

$$* V = k \lambda \ln \left| \frac{0+L}{L} \right|$$

**** iiiii) → uniformly charged disc (σ, R) →

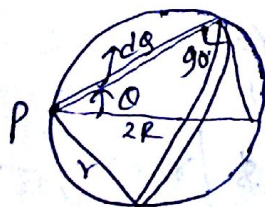
|A| → At centre

$$V = \frac{\sigma R}{2\epsilon_0}$$

|B| → At a point on the axis 'x'

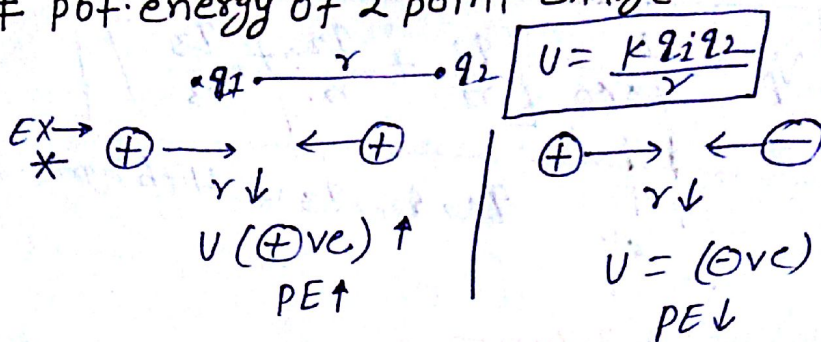
$$V = \frac{\sigma}{2\epsilon_0} \left[\sqrt{R^2 + x^2} - x \right]$$

|C| → At a point on the edge of the disc -



$$V = \frac{\sigma R}{\pi\epsilon_0}$$

pot. energy of 2 point charge.



$$U_{\infty} = \frac{kq_1q_2}{\infty} = 0$$

$$U = \frac{kq_1q_2}{r}$$

* When two point charge are brought close to each other, then PE of system may ↑ or ↓.

Potential energy of system having 'M' point charges - Equal to algebraic sum of PE of all possible pair of 2 charges.

$$\text{no. of pair} = \frac{n(n-1)}{2}$$

Work done by electric field: → conservative field so it perform work then its PE ↓.

$$W_{\text{conser.}} = -\Delta U$$

$$\begin{aligned} W_{\text{electric field}} &= -\Delta U \\ &= -q(\Delta V) \\ &= -q(V_f - V_i) \end{aligned}$$

(charge must use c sign)

* Work in electric field is independent of actual path & Work for closed loop is zero.

Work Energy Relation

$$W_{\text{total}} = \Delta K.E$$

$$W_{\text{external}} + W_{\text{cons}} + W_{\text{non-cons}} = \Delta K.E$$

$$W_{\text{ext}} + W_{\text{cons}} = \Delta K.E$$

$$* W_{\text{ext}} = \Delta K + \Delta U$$

$$K.E = \text{const} \Rightarrow \Delta K.E = 0$$

$$W_{\text{ext}} = ?$$

$$W_{\text{ext}} = \Delta U$$

$$\begin{aligned} &= q\Delta V \\ &= q(V_f - V_i) \end{aligned}$$

If $K.E \neq \text{const}$

$$W_{\text{ext}} = 0$$

$$\Delta K.E + \Delta U = 0$$

$$K + U = \text{const}$$

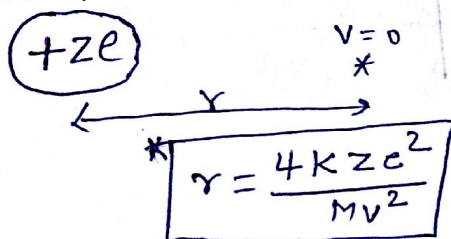
When an α -particle is projected towards a nuclei it can reach up to a closest distance, Higher the $K.E$ of incident α -particle closer to it is to nucleus, success is life analogous to it more effort you put in closer to success.

COME

COME

2016
ATPMT
concept

An α -particle is thrown from with velocity 'v' towards nucleus of atomic no. 'Z' calculate closest distance of approach?



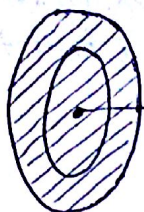
α -particle (+2e)

$$\text{Here } r \propto \frac{1}{m}$$

$$\& r \propto \frac{1}{v^2}$$

ATPMT
2016

Potential at a distance 'x' from axis :-

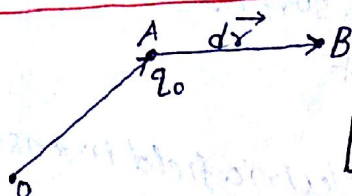


$x = \infty$ $(-q)$

$$V_0 = \sqrt{\frac{qQ}{4\pi\epsilon_0 m R(n+1)}}$$

$$T = 2\pi \sqrt{\frac{R^3 \eta (n+1) m}{2kqQ}}$$

Relation b/w $\vec{E}$ & V :-



"displaced very slowly by Ext. agent"

$$dV_{AB} = -\vec{E} \cdot d\vec{r}$$

NOTE $\Rightarrow$

|| $\rightarrow$ To find potential difference b/w point A & B

$$\int_A^B dV_{AB} = - \int_A^B \vec{E} \cdot d\vec{r}$$

$$V_{AB} = V_B - V_A = \int_A^B \vec{E} \cdot d\vec{r}$$

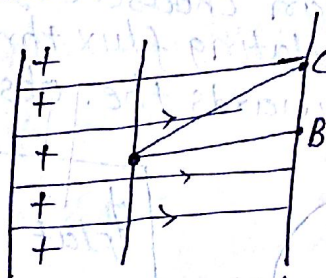
|| $\rightarrow$ To find direction & magnitude of $\vec{E}$

$$dv = -\vec{E} \cdot d\vec{r}$$

$$dv = -E dr \cos \theta$$

$$-\frac{dv}{dr} = E \cos \theta$$

$$\left| -\frac{dv}{dr} \right|_{\max} = E$$



- * Electric field is always directed from a point at higher potential to the point at lower potential.
- * Direction of electric field is along the line where rate of $\downarrow$ of potential (i.e. $-\frac{dv}{dr}$) is $\max$.

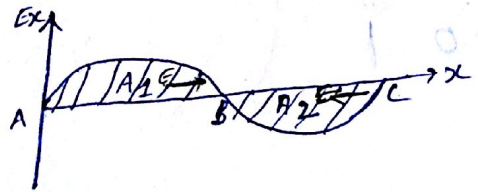
calculating potential from $E-x$ curve

$$E_x = -\frac{dv}{dx}$$

$$\Delta V = -\int E_x \cdot dx \approx \text{Area under } E_x-x \text{ curve.}$$

$$V_{AB} = V_B - V_A = -A_1$$

$$V_{BC} = V_C - V_B = +A_2$$



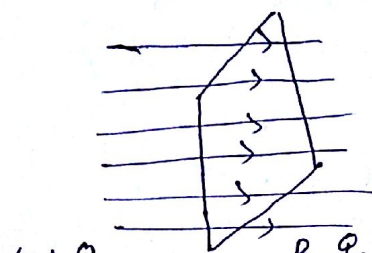
Equipotential surface $\rightarrow$ It is the surface where pot at every point is const. It can be spherical, cylindrical or, plane surface.
 * Electric field is always $\perp$ to the equipotential surface. ($E \perp dv$)
 * Work done is moving a charge ($\oplus ve / \ominus ve$) b/w two point on equipotential surface is always zero.
 $W = q(V_f - V_i)$

$$\therefore V_f = V_i$$

$$W = 0$$

Electric Flux (ϕ) $\rightarrow$ Measure of no of lines passing through a surface.

* Flux going into the surface is consider as $\ominus ve$ & coming out of the surface is consider as $\oplus ve$.



(A) $\ominus$
Flux is going into the surface

(B) $\oplus$
Flux is coming out of surface.

$$d\phi = \vec{E} \cdot d\vec{s}$$

$\vec{E}$ = Electric field intensity.

Area vector ($d\vec{s}$) -



$$d\vec{s} = (ds) \hat{n}$$

$\hat{n}$ = a unit vector $\perp$ to the small area ds

* There can be two directions of $\hat{n}$ & theoretically we can choose any one of them but while calculating flux through a surface $\hat{n}$ is taken towards the observer.

$E \rightarrow$
 E



Hollow Hemisphere

$$\phi_{\text{flat}} = -E\pi R^2$$

$$\phi_{\text{curved}} = E\pi R^2$$

$$\phi_{\text{total}} = 0$$

$$\phi_{\text{bowl}} = \frac{q}{\epsilon_0}$$

NOTE $\rightarrow$ * To find electric flux either we can find area $\perp$ to $\vec{E}$ or, we can find component of $\vec{E}$ $\perp$ to the given Area. [remember area in a case should be plane surface.]
 * If a closed surface is kept in a uniform electric field or, if it does not contain any charge, the total flux passing through it is always zero.

$$\vec{E} \quad \text{[Diagram of a rectangular box]} \quad \phi_{\text{total}} = 0$$

The statement of Gauss law:-

"Net flux passing through any closed surface is equal to charge enclosed by it divided by ϵ_0 "

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}$$

Surface integral = Integration of complete surface.

NOTE →

1a) → * Net flux is always calculated due to charges inside the surface by while integrating $\vec{E} \cdot d\vec{s}$ the $\vec{E}$ at the surface is due to all the charges in that system.

1b) → * With the help of Gauss law, we can find electric field due to the some charged system but they are very limited.

* Angle b/w $\vec{E}$ & $d\vec{s}$ at every point on gaussian surface should be same.

* Magnitude of $\vec{E}$ should be same through out the surface.

* ϕ_{net} doesn't depend on size of body.

Application of Gauss Law:-

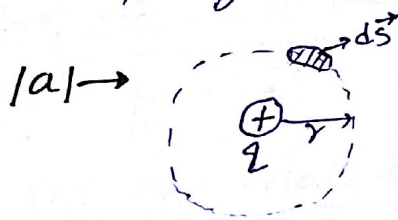
Electric field due to some symmetric charge distribution.

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}$$

$$\oint E ds \cos \theta = \frac{q_{in}}{\epsilon_0}$$

* Magnitude 'E' at every point on surface must be same.

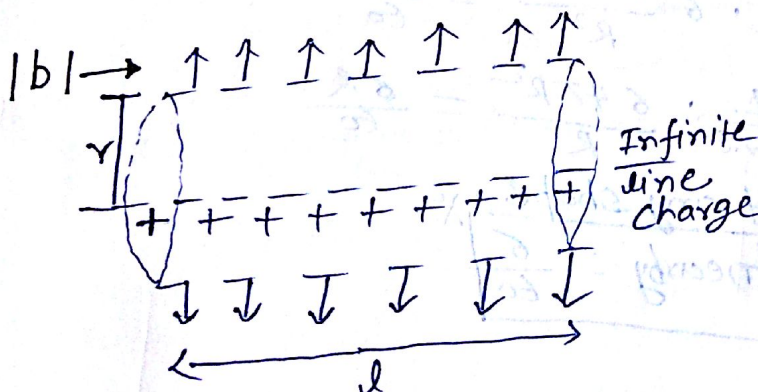
* Angle b/w $\vec{E}$ & $d\vec{s}$ should same.



$$\phi = \vec{E} \cdot d\vec{s} = \oint E ds = E \oint ds = E \times 4\pi r^2$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$E = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{q}{r^2}$$

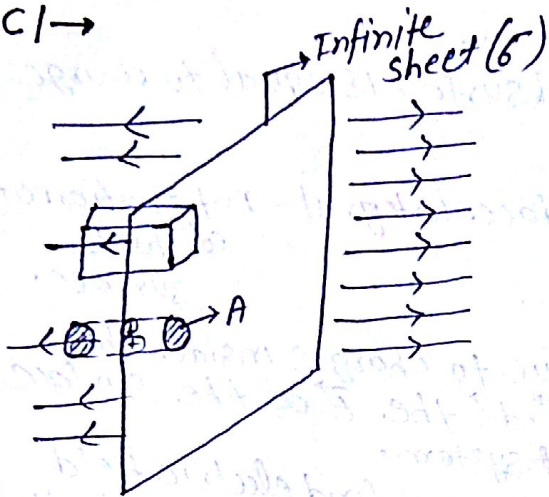


$$\oint \vec{E} \cdot d\vec{s} = E \oint ds = E [2\pi r l]$$

$$= \frac{q_{in}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

|C| →



$$\oint \vec{E} \cdot d\vec{S} = E \oint ds = E(2A)$$

$$Q_{in} = \sigma A$$

* Infinite/long wire: → $E = \frac{2K\lambda}{r}$

* Non conducting wire → $E = \frac{\sigma}{2\epsilon_0} \propto r^0$

* Conducting plate → $E_{outside} = \frac{\sigma}{\epsilon_0}$, $E_{inside} = 0$

All conducting [solid/hollow] & Hollow non conducting sphere

r → distance of observer point from the centre of sphere.



Position	E	V
$r > R$	KQ/r^2	KQ/r
$r = R$	KQ/R^2	KQ/R
$r < R$	0	KQ/R
$r = 0$	0	KQ/R

NOTE →

$E \propto \sigma \rightarrow \sigma = \frac{Q}{4\pi R^2}$

$Q = \sigma (4\pi R^2)$

* $E_{surface} = \frac{KQ}{R^2} = \frac{1}{4\pi\epsilon_0} \cdot \sigma \frac{4\pi R^2}{R^2} = \frac{\sigma}{\epsilon_0}$

* $V_{surface} = \frac{KQ}{R} = \frac{1}{4\pi\epsilon_0} \cdot \sigma \frac{4\pi R^2}{R} = \frac{\sigma R}{\epsilon_0}$

For any conductor of any shape *

$$E_{outside/nearby} = \frac{\sigma}{\epsilon_0}$$

EX → * $E_{\text{surface}} = \frac{kq}{R^2} \propto \frac{1}{R^2}$ (If $q = \text{const}$)

* $E_{\text{surface}} = \frac{\sigma}{\epsilon_0} \propto R^0$ (If $\sigma = \text{const}$)

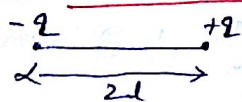
* $E_{\text{surface}} = \frac{V}{R} \propto \frac{1}{R}$ (If $V = \text{const}$)

solid non-conductor sphere:-



Position	E	V
$r > R$	kq/r^2	kq/r
$r = R$	kq/R^2	kq/R
$r < R$	$\frac{kq}{R^3} r$	$\frac{kq(3R^2 - r^2)}{2R^3}$
$r = 0$	0	$\frac{3}{2} \left(\frac{kq}{R} \right)$

Electric dipole :- Two point of same magnitude & opposite nature at small separation.



$2l \rightarrow$ length vector
direction $(-q \text{ to } +q)$

I → Dipole moment (P) :-

$P = \text{Charge} \times \text{length of Dipole}$

$$P = q \cdot 2l$$

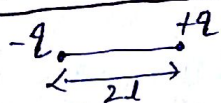
$$\vec{P} = q(2\vec{l})$$

vector = $(-q \text{ to } +q)$

unit = Cxm

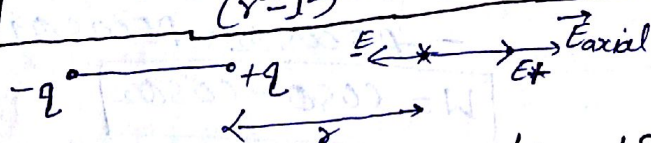
1 Debye(D) = $303 \times 10^{-30} \text{ Cxm}$

II → Electric field due to Dipole

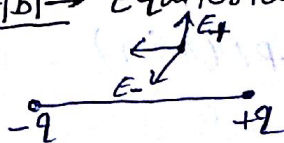


case -a) → Axial / Longitudinal / $\tan A$ / End on position

$$E_{\text{axial}} = \frac{2KPr}{(r^2 - l^2)^2} = \frac{2Kp}{r^3} \text{ (Along P direction)}$$

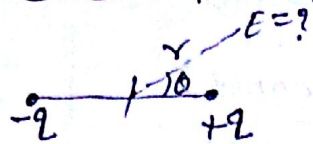


case -b) → Equatorial / Transverse / Broad side / $\tan \beta$ →



$$E_{\text{equatorial}} = \frac{Kp}{(r^2 + l^2)^{3/2}} = \frac{Kp}{r^3} \text{ (opp. direction)}$$

case-|E| → General point (r, θ) :-



$$E = \frac{kP}{r^3} \sqrt{1+3\cos^2\theta}$$

Angle b/w E & P

$$0 + \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

From same distance 's' in case of dipole $\frac{E_{axis}}{E_{equatorial}} = ?$

$$\frac{2kP/r^3}{kP/r^3} = 2:1 \text{ approx}$$

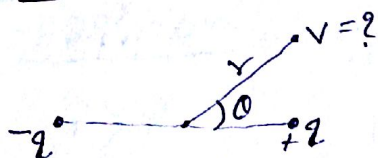
$$\frac{2kP/r^3}{-kP/r^3} = -2:1 \text{ approx}$$

NOTE

- * In axial point dipole the electric field direction in the direction of net dipole.
- * In equatorial dipole electric field direction in opposite to the direction of net dipole.

* $E_{dipole} \propto \frac{1}{r^3}$, $E_{point\ charge} \propto \frac{1}{r^2}$, $E_{long\ wire} \propto \frac{1}{r}$, $E_{sheet} \propto r^0$

III → Potential due to dipole →

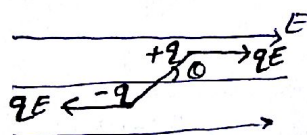


$$V = \frac{kP\cos\theta}{r^2 - a^2\cos^2\theta} = \frac{kP\cos\theta}{r^2}$$

|a| → $\theta = 0$, $V = \frac{kP}{r^2}$

|b| → $\theta = 90^\circ$
 $V_{eq} = 0$

IV → Behaviour of dipole in ext. uniform field: →



* $F_{net} = 0$ (no translational motion)

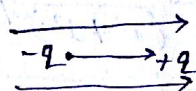
* $\tau = PE\sin\theta$
 $\vec{\tau} = \vec{P} \times \vec{E}$

* $U = -PE\cos\theta$
 $U = -\vec{P} \cdot \vec{E}$

* $W_{\theta_1 \rightarrow \theta_2} = \Delta U$
 $= U_{\theta_2} - U_{\theta_1}$
 $= -PE\cos\theta_2 - (-PE\cos\theta_1)$

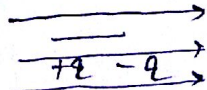
$W = \cos\theta_1 - \cos\theta_2$

case-|a| → If $\theta = 0^\circ$



$f = 0$ | $U = -PE$ (min)
 $\tau = 0$

case-|b| → If $\theta = 180^\circ$



$f = 0$
 $\tau = 0$
 $U = +PE$

NOTE → In a stable equilibrium dipole is given small angular disp. & it perform angular SHM with time period

$$T = 2\pi \sqrt{\frac{I}{PE}} \quad I = M \cdot O \cdot I$$

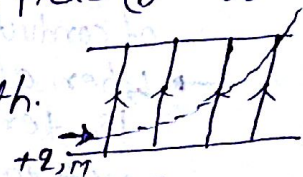
BIG DROP $\rightleftharpoons$ n SMALL DROPS

i) $R_{Big} = n^{1/3} r_{small}$	iv) $E_{Big} = n^{1/3} E_{small}$
ii) $Q_{Big} = n q_{small}$	v) $C_{Big} = n^{1/3} C_{small}$
iii) $\sigma_{Big} = n^{1/3} \sigma_{small}$	vi) $V_{Big} = n^{2/3} V_{small}$

AIPMT 2005

* path of charge particle in uniform electric field ($g = \text{negligible}$)

$y = \left(\frac{qE}{2mv^2} \right) x^2$ → Deviation.
 $y \propto x^2$ → Parabolic path.

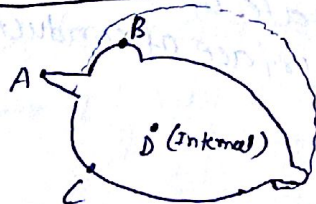


conductor of Irregular shape: → (R → Radius of curvature)

* $Q \propto R$ * $\sigma \propto \frac{1}{R}$ * $E = \frac{\sigma}{\epsilon_0} \propto \frac{1}{R}$ * $V \propto R^0$
 [conductors are equipotential surface]

AIIMS

A Metal Body: →



* $R \Rightarrow R_C > R_B > R_A$

* $Q \Rightarrow Q_C > Q_B > Q_A > Q_D = 0$

* $\sigma \Rightarrow \sigma_A > \sigma_B > \sigma_C$

* $E \Rightarrow E_A > E_B > E_C > E_D = 0$

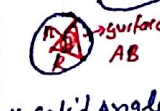
* $V \Rightarrow V_A = V_B = V_C = V_D$

Solid Angle (Ω)

- Plane Angle (θ), Radian measured in 2-D
- Solid angle (Ω), Steradian measured in 3-D



* Angle = $\frac{\text{arc}}{\text{radius}}$
 complete plane Angle $\frac{2\pi R}{R} = 2\pi$

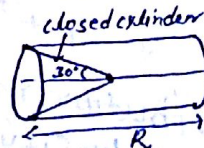


* Solid Angle = $\frac{\text{Area of surface}}{R^2}$
 * complete Solid Angle = $\frac{\pi R^2}{R^2} = \pi$

Application of solid Angle (Ω)

$\Phi_{total} = \frac{q}{\epsilon_0} \cdot 4\pi$
 Flux through per unit solid angle = $\frac{q}{4\pi\epsilon_0}$
 $\cos\alpha = \frac{x}{\sqrt{R^2+x^2}}$ * $\Omega = 2\pi \left[1 - \frac{x}{\sqrt{R^2+x^2}} \right]$

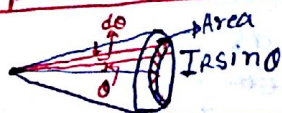
* $\Phi_{\text{through disc}} = \frac{q}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{R^2+x^2}} \right]$



$\Omega_{\text{curved}} = 4\pi - 2\Omega_{\text{flat}} = 4\pi \left[1 - \frac{1}{\sqrt{3}} \right]$
 $\Omega_{\text{flat}} = 2\pi (1 - \cos\theta) = 2\pi (1 - \frac{1}{\sqrt{3}})$

$\Phi_{\text{curved}} = \frac{q}{4\pi\epsilon_0} \times 2\sqrt{3}\pi = \frac{q\sqrt{3}}{2\epsilon_0}$

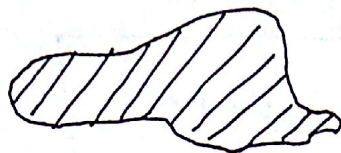
* # Relation b/w plane Angle & solid Angle



* $\Omega = 2\pi (1 - \cos\alpha)$

CONDUCTOR

[METAL]



It has infinite no. of free e^- which can move inside the volume or, on the vol. or, on the surface if external force is applied, they can not leave conductor.

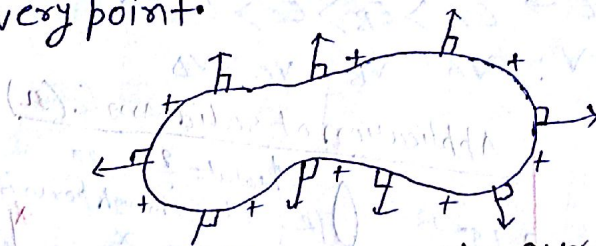
Concept of Electrostatic Equilibrium

Suppose by some mechanism an excess charge $+Q$ is given to a conductor.

- To gain min. energy state, all the charge come on surface of conductor.
- When all the charge come in state of rest, this equilibrium is termed as electrostatic equilibrium.
- Net force on each charge along the surface become zero.

In State of Electrostatic equilibrium

- * Electric field along the surface is zero as $\vec{F}_{\text{net surface}} = 0$.
- * Electric potential of whole body becomes same.
- * Electric field inside the body of conductor become zero.
 $\vec{E}_{\text{inside}} = 0$
- * It can be assumed as lowest energy state of conductor.
- * Electric field lines start $\perp$ from the surface of conductor at every point.



Case I ⇒ Electric field near the surface of conductor.

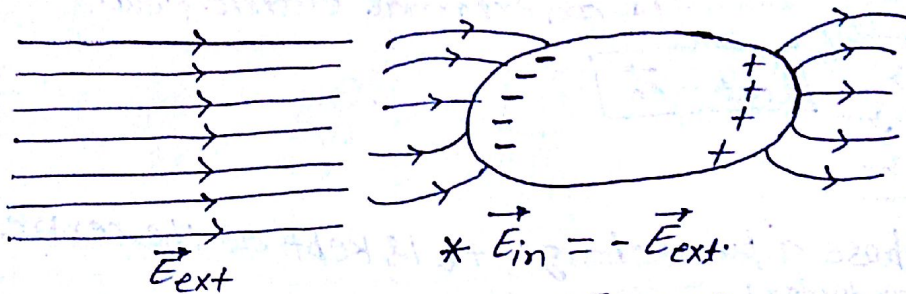
- * Charge distribution may not be uniform.
- * Surface charge density at different points will be different.

$$\oint \vec{E} \cdot d\vec{S} = E \cdot dS = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E dS = \frac{\sigma dS}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0} \vec{n}} \quad \perp \text{ to surface.}$$

case II $\Rightarrow$ conductor left in an external electric field



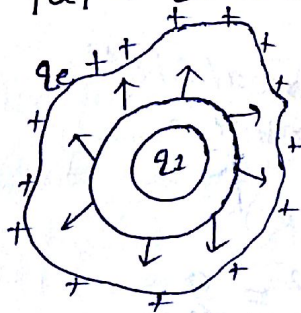
$$* \vec{E}_{in} = -\vec{E}_{ext}$$

$$* E_{net} = 0$$

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case III $\Rightarrow$ Conductor Having cavity $\rightarrow$

Ia) $\rightarrow$ Excess charge given but no charge in cavity $\rightarrow$



* under electrostatic condition

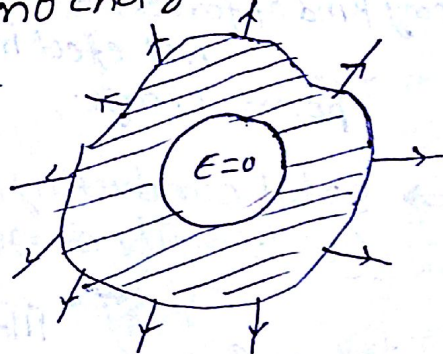
$$V_{conductor} = \text{const.}$$

$$E_{inside} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\epsilon_0} = \frac{q_1}{\epsilon_0}$$

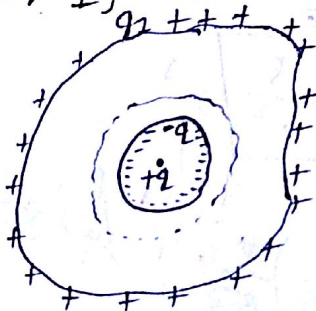
$$\boxed{\vec{E} = 0}$$

$$\dots \boxed{q_{in} = 0}$$



Ib) $\rightarrow$ A point charge is kept inside the cavity $\rightarrow$

II) $\rightarrow$ If no. charge given to conductor $\rightarrow$



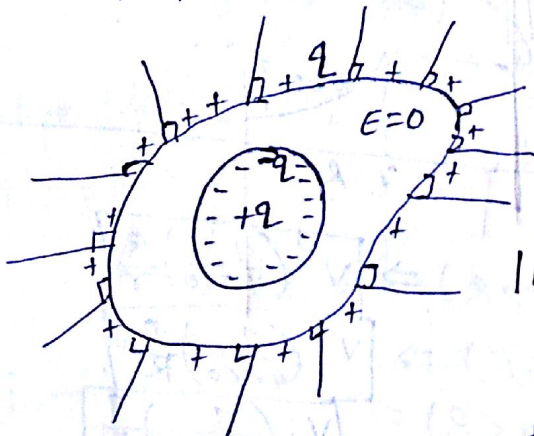
$$\oint \vec{E} \cdot d\vec{s} = 0$$

$$\frac{q - q_1}{\epsilon_0} = 0$$

$$\Rightarrow \boxed{q_1 = q}$$

Electric field inside cavity at a distance r from O

$$\Rightarrow \boxed{E = \frac{kq}{r^2}}$$

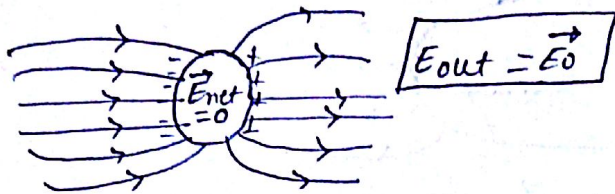


III) $\rightarrow$ If additional charge is also given to the conductor.



$$\boxed{Net = +q + q}$$

**** Case IV $\Rightarrow$ Electrostatic shielding $\Rightarrow$ Suppose a conductor is kept in an external electric field.**



Now, suppose a point charge $+q$ is kept at the centre of conductor

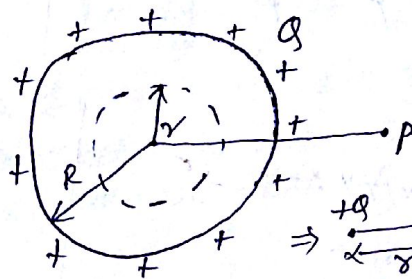
$(+q)$ $\boxed{E_{ext} = E_0}$

AIIMS

$Q \rightarrow$ Will $+q$ experience any force?

Ans $\rightarrow$ If charge $+q$ is placed inside conductor, it will not experience any kind of force, it is known as Electrostatic shielding. Net effect inside conductor will be only due to point charge.

Case V $\Rightarrow$ Solid conducting sphere or, Hollow spherical shell (uniformly charged) Electric field intensity:



iii $\rightarrow$ For inside point ($r < R$)

$$\oint \vec{E} \cdot d\vec{S} = E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\boxed{E_{inside} = 0}$$

iii $\rightarrow$ For outside point ($r > R$)

$$\oint \vec{E} \cdot d\vec{S} = E \times 4\pi r^2 = \frac{Q}{\epsilon_0}$$

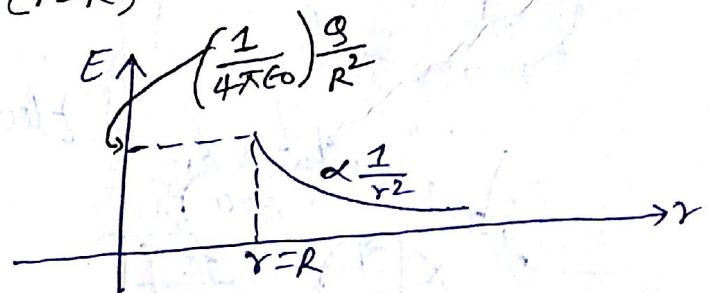
$$\boxed{E_{outside} = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{Q}{r^2}}$$

AIIMS

iiii $\rightarrow$ on surface (just outside) ($r = R$)

$$\boxed{E = \left(\frac{1}{4\pi\epsilon_0}\right) \left(\frac{Q}{R^2}\right)}$$

Just inside $\Rightarrow (r = R)$
 $\boxed{E = 0}$



Electric potential

i $\rightarrow$ For outside point ($r > R$) $\Rightarrow \boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{Q}{r}}$

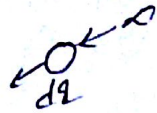
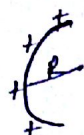
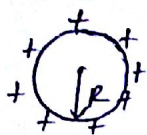
ii $\rightarrow$ For on surface ($r = R$) $\Rightarrow \boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{Q}{R}}$

iii $\rightarrow$ For inside point ($r < R$) $\Rightarrow \boxed{V = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{Q}{R}}$

$= \text{const} = V_{\text{surface}}$

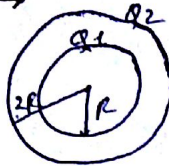


Self potential energy of conducting shell $\Rightarrow$



$$W_{ext} = \frac{kq}{2R} = \frac{q^2}{8\pi\epsilon_0 R} = U_{self}$$

EX $\rightarrow$



$U_{system} = ?$

$$U_{system} = U_{self} + U_{interaction}$$

$$U_{self} = \frac{Q_1^2}{8\pi\epsilon_0 R} + \frac{Q_2^2}{8\pi\epsilon_0 (2R)}$$

$$U_{interaction} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1 Q_2}{2R} \right)$$

Uniformly charged NON-conducting solid sphere :-

i) $\rightarrow$ Electric field ($E(r)$)

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qr}{R^3} \quad \leftarrow || \rightarrow r < R$$



$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\oint \vec{E} \cdot d\vec{S} = E \cdot 4\pi r^2$$

$$Q_{in} = \rho \times \frac{4}{3}\pi r^3$$

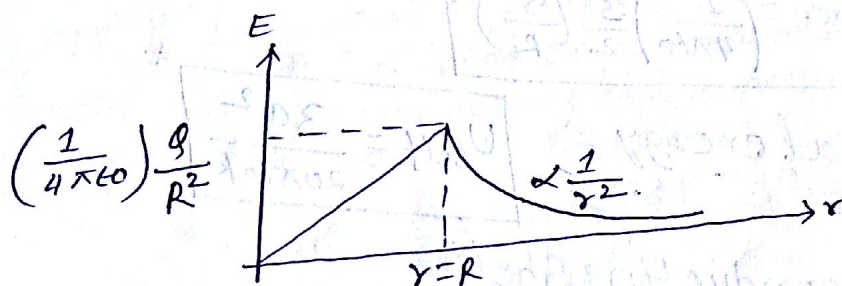
ii) $\rightarrow r = R$

$$E_{surface} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{R^2}$$

iii) $\rightarrow$ For outside points whole charge of sphere can be assumed at centre. ($r > R$)

$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{r^2}$$



ib) $\rightarrow$ Electric potential $V(r) \rightarrow$

ii) $\rightarrow r > R$ (For outside point)

$$V(r) = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{r}$$

iii) $\rightarrow r = R$ (at surface)

$$V_{surface} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{R}$$

iii) $r < R$ (For inside point)



$$-\frac{dv}{dr} = E$$

$$-\int_{V_{\text{surface}}}^{V_{\text{inside}}} dv = \int_{r=R}^{r=r} E \cdot dr$$

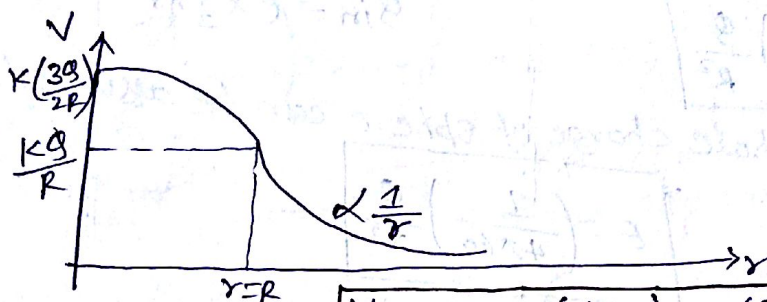
$$V_{\text{inside}} = V_1 + V_2$$

due to point
 $r < r$

due to point
 $r < r < R$

$$V_1 = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q r^3}{R^3}$$

$$V_{\text{inside}} = V_1 + V_2 = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{2R^3} (3R^2 - r^2)$$

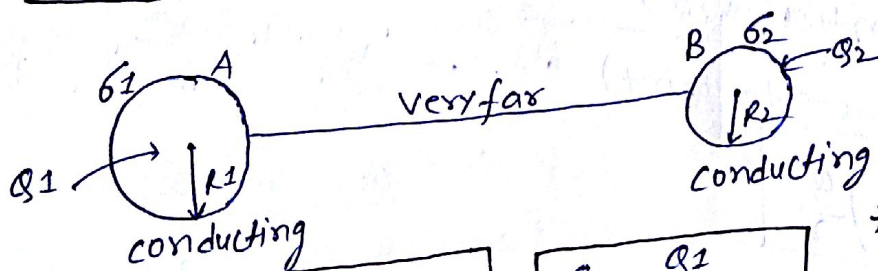


$$V_{\text{centre}} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{3}{2} \left(\frac{Q}{R} \right)$$

|C| $\rightarrow$ self potential energy $\Rightarrow$

$$U_{\text{self}} = \frac{3Q^2}{20\pi\epsilon_0 R}$$

connecting two conducting shells



$$V_A = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q_1}{R_1}$$

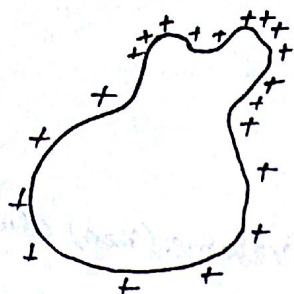
$$V_B = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q_2}{R_2}$$

$$Q_1 = \frac{Q_1}{4\pi R_1^2}$$

$$Q_2 = \frac{Q_2}{R_2^2}$$

$$Q_1 R_1 = Q_2 R_2$$

$$Q \propto \frac{1}{\text{radius of curvature}}$$



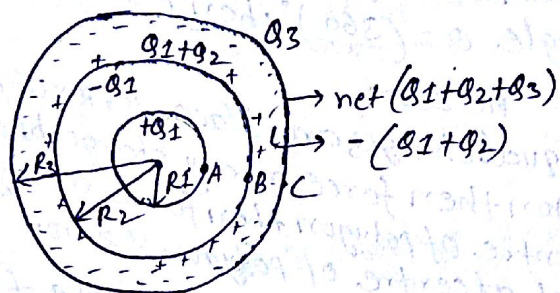
It is clear that at sharp edges surface charge density become too high.

$$\sigma = \frac{1}{\text{radius of curvature}}$$

AIIMS

****** If Electric field just outside the conductor become greater than $3 \times 10^6 \text{ V/m}$, break down of air molecule near conductor start which is commonly known as 'Corona Discharge' (Ionisation of air molecules)

Potential calculating in conducting shells

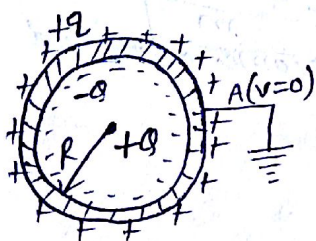


$$V_A = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{R_1} + \frac{Q_2}{R_2} + \frac{Q_3}{R_3} \right)$$

$$V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{R_2} + \frac{Q_2}{R_2} + \frac{Q_3}{R_3} \right)$$

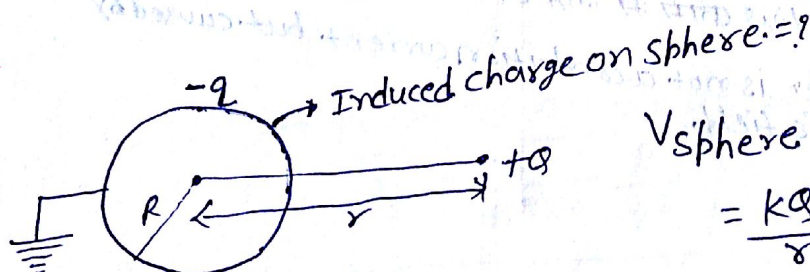
$$V_C = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1 + Q_2 + Q_3}{R_3} \right)$$

NOTE →



- * By convention potential of earth is assumed to be zero.
- * Two points which are earthed can be connected by conducting wire. ($V = \text{const} = 0$)
- * The point or body connected to earth can receive or send desired charge accordingly charge conservation law will not hold.

$$V_A = \frac{kQ}{R} + \frac{k(Q-Q)}{R} = 0 \Rightarrow Q_1 = 0$$



$$V_{\text{sphere}} = V_{\text{centre}}$$

$$= \frac{kQ}{r} - \frac{kQ_1}{R} = 0$$

$$Q_1 = \frac{QR}{r}$$

Point from question

AIMS * $F > Ab \cdot C > \text{coulumb} > \text{stat. coulumb}$

* Faraday $> Ab \cdot C > \text{coulumb} > \text{stat. coulumb}$

* $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $m_0 \Rightarrow \text{rest mass.}$

* Rest mass of photon zero.

* When a body is charge (either (+ve)ly or (-ve)ly) its volume (size) always

↑ & density always ↓

* In all type of RKN total charge of system remain const.

* Total no. of ions in a universe is const → wrong.

* $1 \leq \epsilon_r \leq \infty$

* $K = [m^2 L^3 T^{-4} A^{-2}] \rightarrow \text{kappa particle}$

* When dielectric medium place b/w charge then electric force direction.

* When metal is placed b/w charge then electric force become zero.

#

0	F_R
0°	$2F$
60°	$\sqrt{3}F$
90°	$\sqrt{2}F$
120°	F
180°	0

* If an equal forces are acting at an angle $\theta = \left(\frac{360}{N}\right)^\circ$ then resultant will be zero.

* If equal charges are place vertex of regular polygon then force on any charge placed at centre of polygon then force on any charge placed at centre of polygon is zero.

* A charge particle (q, m) is release from rest in an uniform E field then $K \cdot E$ after time 't' $\Rightarrow K \cdot E = \frac{1}{2} \frac{q^2 E^2 t^2}{m}$

* When temp. of dielectric medium ↑ then molecule of medium become disturbed so net induced electric field ↓ that's why ϵ_r also ↓

temp ↑ $\Rightarrow E_{ind} \downarrow$ $\left(\frac{\epsilon_0}{\epsilon_r} = \epsilon_0 - E_{ind} \right)$

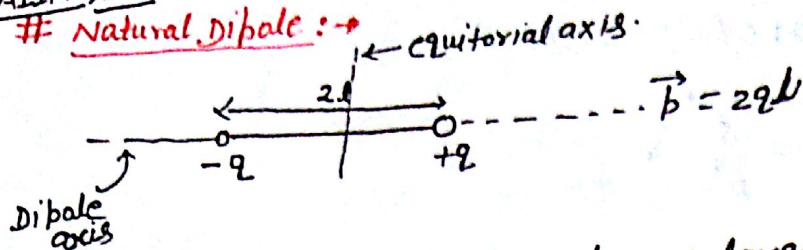
$\downarrow \epsilon_r = \frac{\epsilon_0}{(\epsilon_0 - E_{ind})} \uparrow$

* Hollow sphere. # centre to surface pot. same $\frac{Q}{4\pi\epsilon_0 r^2}$ centre to surface pot. calculate $\frac{Q}{4\pi\epsilon_0 r^2}$ radius कास में जोते है। नत नि दूरी !!

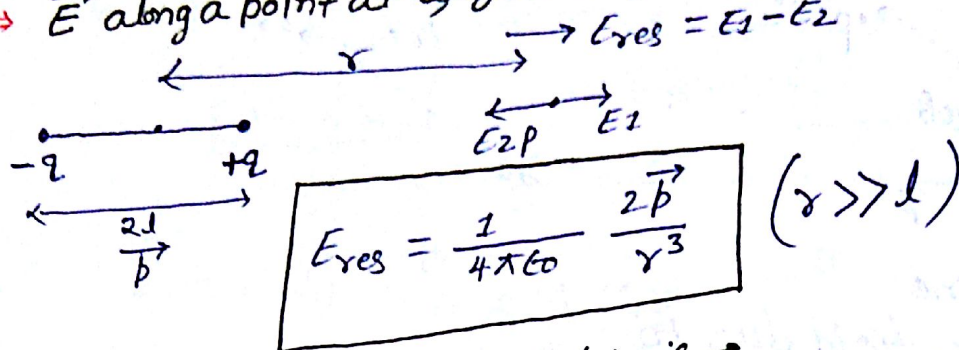
* Displacement current is same as is not a conduction current but caused by time varying electric field.

* $\frac{Q}{4\pi\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_0 r^2}$

Natural Dipole :-

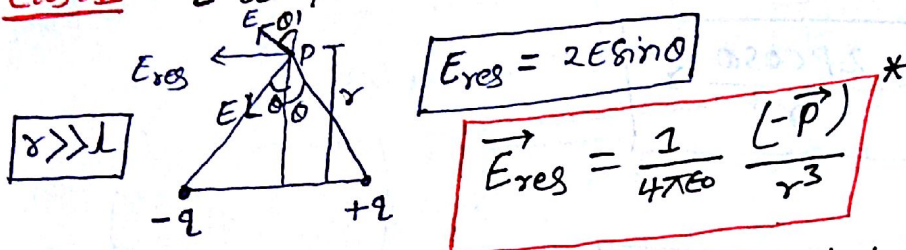


Case I $\Rightarrow$ $\vec{E}$ along a point at a large distance ' r ' on dipole axis $\rightarrow$



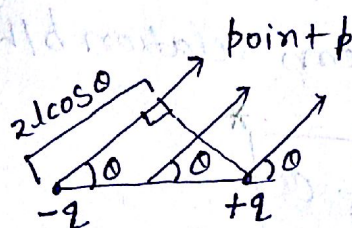
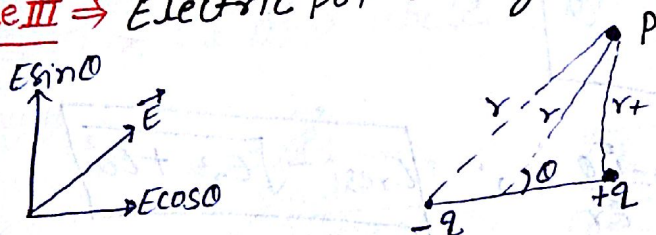
$$E_{res} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3} \quad (r \gg l)$$

Case II $\Rightarrow$ $\vec{E}$ at point on equatorial axis $\rightarrow$



$$\vec{E}_{res} = \frac{1}{4\pi\epsilon_0} \frac{(-\vec{p})}{r^3} \quad *$$

Case III $\Rightarrow$ Electric pot. at a general point $(r, \theta) \rightarrow$



* If point P at very large distance $\rightarrow$

$$= \frac{q(2l \cos \theta)}{4\pi\epsilon_0 r^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{p \cos \theta}{r^2} \right)$$

* At a point on dipole axis $\rightarrow$

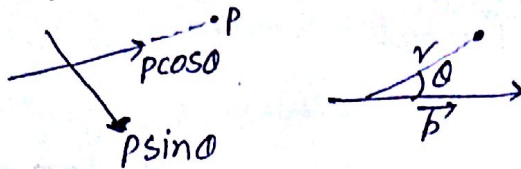
$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$$

* At a point on equatorial axis $\rightarrow$

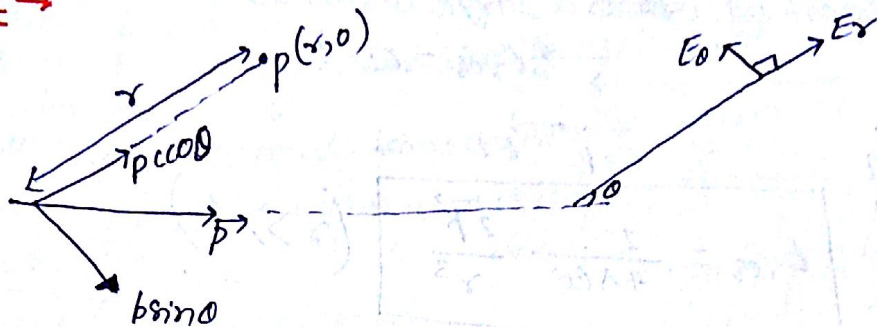
$$\theta = 90^\circ$$

$$V = 0 \quad *$$

NOTE → Dipole as in form of components



Case IV →



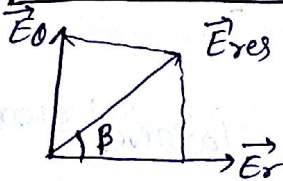
Electric field due to

|a| → $p \cos \theta$

$$\vec{E}_r = \frac{1}{4\pi\epsilon_0} \frac{2p \cos \theta}{r^3} \hat{r}$$

|b| → $p \sin \theta$

$$E_\theta = \frac{1}{4\pi\epsilon_0} \frac{p \sin \theta}{r^3} \hat{\theta}$$



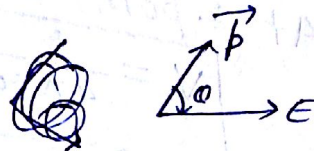
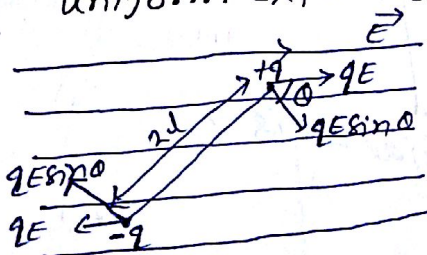
$$\tan \beta = \frac{E_\theta}{E_r}$$

$$E_{res} = \sqrt{E_r^2 + E_\theta^2}$$

NOTE ⇒ In case of polar co-ordinate system, relation b/w $\vec{E}$ & V can be given by

$$\vec{E} = - \frac{\partial V}{\partial r} \hat{r} - \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta}$$

Case V ⇒ Net force & torque acting on dipole kept in uniform external $\vec{E}$.



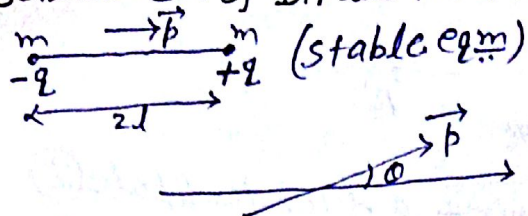
$$\vec{F}_{net} = 0$$

calculating $\vec{\tau}$ about centre of mass

$$\tau = 2 (qE \sin \theta) a \quad \left| \quad \begin{aligned} \tau &= pE \sin \theta \\ \tau &= \vec{p} \times \vec{E} \end{aligned} \right.$$

$$\tau = \vec{p} \times \vec{E}$$

Case VII $\Rightarrow$ oscillation of Dipole in uniform $\vec{E} \rightarrow$



Linear oscillation

$$\omega = \sqrt{\frac{k}{m}}$$

Angular oscillation

$$\omega = \sqrt{\frac{k}{I}}$$

$$\tau = -PE \sin \theta$$

$\therefore \theta$ is very small

$$\sin \theta \approx \theta$$

$$\text{Time period} = T = 2\pi \sqrt{\frac{I}{PE}}$$

$$I = ml^2 + ml^2 = 2ml^2$$

Case VIII $\Rightarrow$ PE of a dipole in uniform External field: $\rightarrow$

$\vec{p}$ pointing right, charges -q and +q aligned horizontally.

Stable. $PE = U = \text{Min}$

charges +q and -q aligned horizontally, $\vec{p}$ pointing right.

unstable $PE = U = \text{Max}$

charges +q and -q aligned vertically, $\vec{p}$ pointing right.

$U_{\text{min}} < U < U_{\text{max}}$

Work done on dipole in rotating it from stable position is stored in the form of PE of dipole.

suppose from general position $\theta = \theta$; dipole is rotated very slowly through an angle ' $d\theta$ '

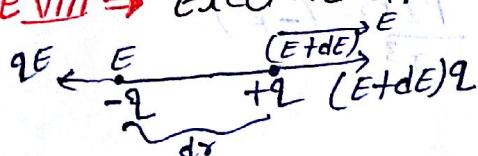
$$U = -PE \cos \theta$$

$$U = -\vec{p} \cdot \vec{E}$$

If dipole is rotated from $\theta = \theta_1$ to $\theta = \theta_2$ work done by ext agent = $W_{\text{ext}} = U_f - U_i$

$$W_{\text{ext}} = PE (\cos \theta_1 - \cos \theta_2)$$

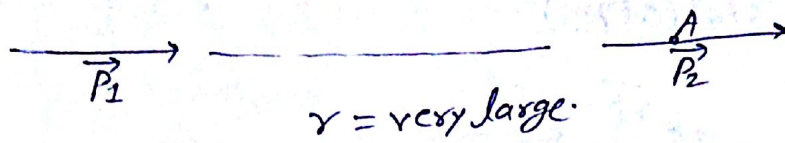
Case VIII $\Rightarrow$ Electric dipole in non-uniform electric field $\rightarrow$



Net force = $F = q(dE)$
 $= q(dx) \left(\frac{dE}{dx} \right)$

$$F = p \left(\frac{dE}{dx} \right)$$

case IX $\Rightarrow$ Force of interaction b/w two dipoles $\rightarrow$



Method (1) Electric field at point A due to dipole (1)

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{2P_1}{r^3}$$

$$\frac{dE_1}{dr} = \frac{1}{4\pi\epsilon_0} \frac{(-6)P_1}{r^4}$$

* Force acting on dipole (2)

$$F = P_2 \left(\frac{dE_1}{dr} \right) = -\frac{1}{4\pi\epsilon_0} \frac{6P_1 P_2}{r^4}$$

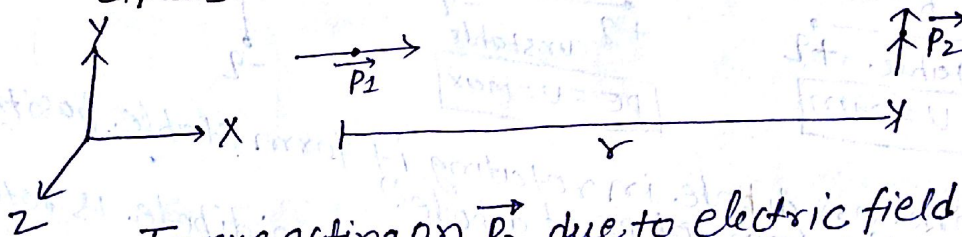
NOTE $\Rightarrow \frac{dE}{dr} \rightarrow \ominus$ ve $\leftarrow$ sign indicates that force is attractive in nature.

Method (2) PE of Dipole (2) kept in electric field of dipole (1) is given by -

$$U = -P_2 \left(\frac{1}{4\pi\epsilon_0} \right) \frac{2P_1}{r^3}$$

$$\checkmark \text{ Force } = F = -\frac{dU}{dr} = -\frac{1}{4\pi\epsilon_0} \frac{6P_1 P_2}{r^4}$$

case X $\Rightarrow$ Torque & potential energy (PE) of interaction b/w two dipoles $\Rightarrow$



Torque acting on $\vec{P}_2$ due to electric field of $\vec{P}_1$

$$\vec{P}_1 = P_1 \hat{i}, \vec{E}_1 \text{ at distance } r \Rightarrow \vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{2P_1}{r^3} \hat{i}$$

$$\vec{P}_2 = P_2 \hat{j} \quad \vec{\tau}_{P_2} = \vec{P}_2 \times \vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{2P_1 P_2}{r^3} (-\hat{k})$$

$$\vec{E}_1 \text{ at distance 'r'} \Rightarrow \vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{2P_2}{r^3} (-\hat{j})$$

(due to $\vec{P}_2$ at $\vec{P}_1$)

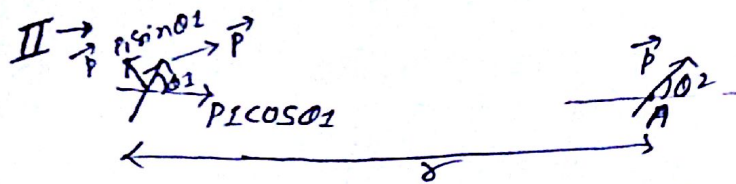
$$\vec{P}_1 = P_1 \hat{i}$$

$$\vec{\tau}_{P_1} = \vec{P}_1 \times \vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{P_1 P_2}{r^3} (-\hat{k})$$

For General System of Dipoles interaction energy (U) $\Rightarrow$



$$U = -\frac{1}{4\pi\epsilon_0} \frac{2P_1 P_2}{r^3}$$



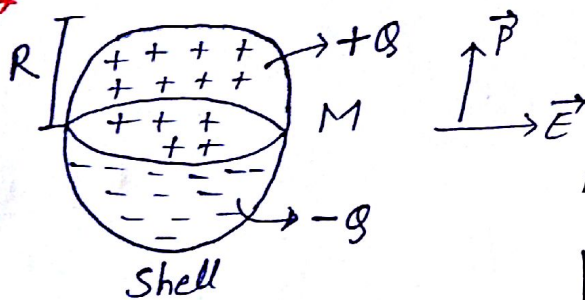
Electric field due to $\vec{P}_1$ at point A

$$\vec{E}_{P_1} = \frac{1}{4\pi\epsilon_0} \frac{2P_1 \cos\theta_1}{r^3} \hat{i} - \frac{1}{4\pi\epsilon_0} \frac{P_1 \sin\theta_1}{r^3} \hat{j}$$

$$\vec{P}_2 = P_2 \cos\theta_2 \hat{i} + P_2 \sin\theta_2 \hat{j}$$

$$* U = -\vec{P}_2 \cdot \vec{E}_{P_1}$$

g $\rightarrow$



If the shell is released from the position shown, then find -

1a) $\rightarrow$ Initial Angular Accn. -

Initial torque

$$\vec{\tau}_i = \vec{P} \times \vec{E} = PE = I\alpha$$

$$I = \frac{2}{3} MR^2$$

$$\alpha = \frac{3PE}{2MR^2} = \frac{3EQR}{2MR^2} = \boxed{\frac{3EQ}{2MR}}$$

1b) $\rightarrow$ Angular Speed when it rotated through $\theta = 90^\circ$ *

$$\boxed{\omega = \sqrt{\frac{2EQR}{I}}}$$